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A MODEL OF INTERACTION OF A VIBROACOUSTIC SYSTEM WITH A RIGID BASE

M. G. Kiselev, V. A. Nifagin, and A. V. Drozdov

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Mathematical description of the kinematic and dynamic parameters of interaction between the links of an open-loop acoustic system of a modernized section of a sawmill is presented. Based on the suggested mathematical model we obtained relations which allow one to judge, by numerical methods, the influence of a vibroshock regime of processing of fragile materials on the main parameters of the process of sawing under different conditions of processing (speed of cutting, static load). The law of motion of the system is analyzed, qualitative conclusions are drawn, and values of the power characteristics are obtained.

At present, mechanical sawing, i.e., separation of blanks into parts, which, however, is characterized by a very low efficiency, is used in manufacture of articles of precious stones and different single crystals. As is known [1, 2], one of the effective means of enhancement of the process, improvement of the accuracy and quality of processing of materials by cutting, is to use the energy of ultrasound through transfer of vibrations to the tool or workpiece being processed. The new equipment for vibroshock mechanical sawing of diamond crystals described below is intended to speed up the process in manufacture of diamonds. Preliminary experiments on the use of ultrasound in order to improve the efficiency and quality of the process of sawing of fragile materials have been conducted. It has been found that enhancement of the process of brittle fracture requires that interaction of the cutting edge of the sawing disk with the material processed occur in a vibroshock regime, i.e., high-frequency shock interaction of them could take place. An original method of processing which employs an acoustic (ultrasonic) open-loop vibrational system has been suggested for realization of the regime mentioned (Fig. 1).

To describe the dynamics of the process of interaction between the open-loop vibroacoustic system and the absolutely rigid base, we use the model (Fig. 2) suggested in [3], which consists of a concentrated mass M loaded by a static (constant) force P_{st} and provided with a vibroelement (whose mass can be neglected) with the characteristic

$$u(t) = f(t) - \frac{R(t)}{c_s}, \tag{1}$$

where $f(t) = A[1 - \gamma(t)\cos \omega_{ac} t]$ for a harmonic ultrasonic vibration at a frequency $\nu_{ac} = \omega_{ac}/2\pi$ and a period $T_{ac} = 1/\nu_{ac}$, $\gamma(t) = a \exp(-bt)$, $a > 0$, $b \geq 0$.

Relation (1) was used in [3] for estimation of the momentum and duration in a single collision between the vibroelement and the base. In particular, when $2c_s A > P_{st}$ the interacting contact is vibroshock, when its duration T_s turns out to be smaller than T_{ac} of ultrasonic vibrations. Therefore, a regime of multiple collisions presupposes a discrete instant of momentum transfer, which is inconvenient for mathematical description of it and makes use of the estimates given in [3] difficult, thus leading to the necessity of summation of a great number of small terms.

In order to overcome the difficulties mentioned and to transform discrete representations to continuous ones, it is worth averaging the momentum transferred in time T_s over the period of ultrasound T_{ac}, i.e., in time T_s the momentum is

$$I_{ac} = \int_{\Gamma} R(t)\, dt .$$

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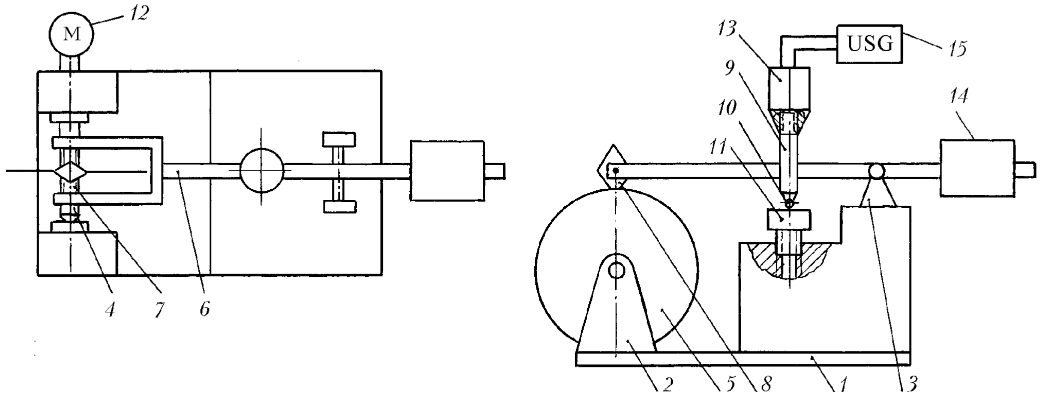

Fig. 1. Schematic of the setup for sawing of diamond crystals: 1) bed; 2) front pair of pillars; 3) rear pair of pillars; 4) spindel; 5) sawing disk; 6) boom; 7) fixation arbors; 8) workpiece processed; 9) concentrator of ultrasonic vibrations; 10) intermediate element; 11) moving platform for delivery of the workpiece; 12) synchronous electric motor; 13) ultrasonic transducer; 14) adjustable counter weight; 15) generator of ultrasonic vibrations.

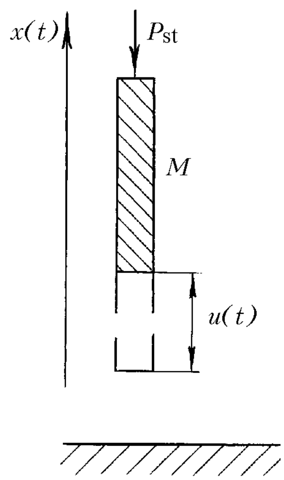

Fig. 2. Model of interaction of the ultrasonic vibrator with the rigid base.

Introducing relation (1), for the force at the end we have

$$I_{ac} = c_s \int_{\Gamma} [f(t) - u(t)]\, dt\,;$$

then in time T_{ac} the averaged (per time unit) momentum is

$$I_m = \frac{c_s}{T_{ac}} \int_{\Gamma} [f(t) - u(t)]\, dt\,.$$

Since $u(t) = x(t)$, on the section of contact Γ ($\alpha_s \le t \le \beta_s$) we obtain

$$I_m = \frac{c_s}{T_{ac}} \int_{\alpha_s}^{\beta_s} [f(t) - x(t)]\, dt\,. \qquad (2)$$

A multiple character of interaction is possible for large values of the mass of the system $M >> I_{ac}/(gT_{ac})$. The duration $\beta_s - \alpha_s$ of a single collision is small compared with T_s. Therefore, the change of the coordinate of the system $x(t)$ on $\alpha_s \le t \le \beta_s$ can be neglected. Assuming $x(t) = x$ at the beginning of the next contact, we can write (2) in the form

$$I_m = \frac{c_s}{T_{ac}} \int_{\alpha_s}^{\beta_s} [f(t) - x]\, dt = \frac{c_s}{T_{ac}} \left[\int_{\alpha_s}^{\beta_s} f(t)\, dt - x(\beta_s - \alpha_s) \right]. \quad (3)$$

From the condition $R(t) \ge 0$ follow the relations for determining α_s and β_s:

$$f(\alpha_s) = x, \quad f(\beta_s) = x, \quad f(t) \ge x \;\text{ when }\; \alpha_s \le t \le \beta_s. \quad (4)$$

Determining $\alpha_s = \alpha_s(x)$ and $\beta_s = \beta_s(x)$ from (4) and substituting in (3), we find the dependence $I_m(x)$ of the mean (per time unit) momentum of interaction of the vibrosystem and the base. Then, in time t the momentum is

$$I(t) = \int_0^t I_m(\tau)\, d\tau, \quad I_m = \frac{dI(t)}{dt}. \quad (5)$$

From (4) we find

$$A[1 - \gamma(\alpha_s) \cos \omega_{ac} \alpha_s] = x, \quad A[1 - \gamma(\beta_s) \cos \omega_{ac} \beta_s] = x,$$

whence

$$Aa \exp(-b\alpha_s) \cos \omega_{ac} \alpha_s = A - x, \quad Aa \exp(-b\beta_s) \cos \omega_{ac} \beta_s = A - x. \quad (6)$$

In this case, the condition of approach of the end to the base up to the distance of vibrocontact $0 \le x \le 2A$ must hold.

Integrating in (3), we have

$$I_m = \frac{dI(t)}{dt} = \frac{c_s}{T_{ac}} \left[\int_{\alpha_s}^{\beta_s} A[1 - \gamma(t) \cos \omega_{ac} t]\, dt - x(\beta_s - \alpha_s) \right] =$$

$$= \frac{c_s}{T_{ac}} \left[(A - x)(\beta_s - \alpha_s) - a_s \frac{A \exp(-b\beta_s)}{b^2 + \omega_{ac}^2} (\omega_{ac} \sin \omega_{ac}\beta_s - b \cos \omega_{ac}\beta_s) + \right.$$

$$\left. + a \frac{A \exp(-b\alpha_s)}{b^2 + \omega_{ac}^2} (\omega_{ac} \sin \omega_{ac}\alpha_s - b \cos \omega_{ac}\alpha_s) \right].$$

Substituting (6) in the last expression and allowing for (3), (5), and the relation $T_{ac}\omega_{ac} = 2\pi$, we obtain

$$\frac{dI(t)}{dt} = \frac{c_s}{\pi} \left[(A - x)(\beta_* - \alpha_*) - a_* \frac{A \exp(-b\beta_*)}{b^2 + \omega_{ac}^2} (\omega_{ac} \sin \omega_{ac}\beta_* - b \cos \omega_{ac}\beta_*) + \right.$$

$$\left. + \frac{aA \exp(-b\alpha_*)}{b^2 + \omega_{ac}^2} (\omega_{ac} \sin \omega_{ac}\alpha_* - b \cos \omega_{ac}\alpha_*) \right], \quad (7)$$

where α_* and β_* are the solutions of (6) which depend on x.

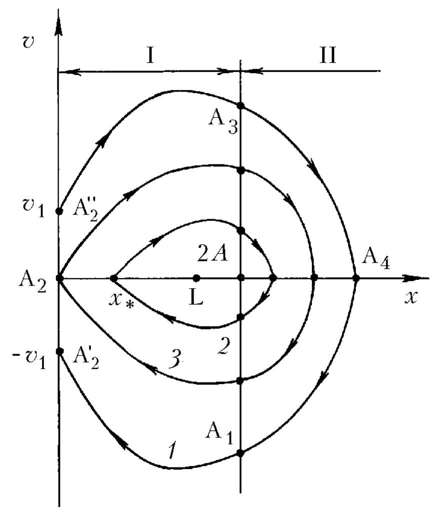

Fig. 3. Phase picture of motion of the system.

In order to simplify (7), instead of harmonic ultrasonic vibration we can consider its square approximation

$$f(t) = \begin{cases} \dfrac{4A}{T_{ac}}(a_0 t^2 + b_0 t), & 0 \le t \le \dfrac{T_{ac}}{2}; \\ \dfrac{4A}{T_{ac}}(a_1 t^2 + b_1 t + c_1), & \dfrac{T_{ac}}{2} < t \le T_{ac}. \end{cases} \tag{8}$$

In this case, dependences (8) must retain both the amplitude value of the displacement of the end $2A$ and the maximum momentum imparted to a contacting body in a harmonic law of ultrasonic vibrations.

From (4) and (8) follows

$$\frac{4A}{T_{ac}}(a_0\alpha^2 + b_0\alpha) = x, \quad \frac{4A}{T_{ac}}(a_1\beta^2 + b_1\beta + c_1) = x. \tag{9}$$

From (9) we find values of α_s and β_s, and from (3) we obtain a formula for the mean force of interaction between the end and the base. This formula is more compact compared to (7).

Generally speaking, various approximations of the function $f(t)$, which after averaging, similar to that done above, lead to different dependences, are admissible. For example,

$$I_m(x) = cAF\left(\frac{x}{2A}\right). \tag{10}$$

The function $F(x/2A)$ decreases monotonically by $x/2A \in [0; 1]$, with $F(0) = 1$ and $F(1) = 0$.

In what follows, we use, in particular, the power dependence of the form

$$I_m(x) = cA\left(1 - \frac{x}{2A}\right)^{\alpha}, \quad \alpha > 0, \tag{11}$$

with the parameters c and α being chosen experimentally.

Relations (7), (10), and (11) hold on the sections $0 \le x \le 2A$ of vibrocontact of the system with the base (Fig. 3, phase of motion I). When $x > 2A$ (phase II), $I_m = 0$ is adopted. At the same time, such a "finite" assignment of the function $I_m(x)$ is not obligatory; it is allowable to use other approximations in the form of smooth functions which are rather close to zero when $x > 2A$. This presentation is more convenient in terms of apparatus, since there is no need to separate the motion into phases.

The momentum imparted by the system to the base can be found at known $x(t)$ by the formula

$$I(t) = \int_0^t I_m [x(\tau)]\, d\tau = cA \int_0^t F\left[\frac{x(\tau)}{2A}\right] d\tau .$$

We consider the dynamics of interaction between the system and the rigid base. To analyze the law of motion $x(t)$ of the system which interacts with the base (Fig. 2), we write the equation of motion

$$M\ddot{x}(t) = -P_{st} + I_m(x) \tag{12}$$

with the initial conditions

$$x(0) = 2A, \quad \dot{x}(0) = -v_0, \tag{13}$$

which correspond to the onset of motion $t = 0$ chosen at the moment when the system enters phase I (Fig. 3), i.e., when the system is at point $x(0) = 2A$.

Solution of (12), (13) with account for (7) is written as

$$x(t) = -\int_0^t G(\tau, \xi)\, \Phi(\xi)\, d\xi + \sum_{k=1}^{2} A_k x_k(t), \tag{14}$$

where

$$G(t, \xi) = \frac{\delta \sin \vartheta \left(\frac{\pi}{2} - t\right) \sin \vartheta\xi}{\vartheta \sin \vartheta};$$

$$\Phi(\xi) = \frac{c_s}{T_{ac}}\left[(A - \xi)(\beta - \alpha) - a\,\frac{A \exp(-b\beta_*)}{b^2 + \omega_{ac}^2}(\omega_{ac} \sin \omega_{ac}\beta_* - b \cos \omega_{ac}\beta_*) + \right.$$

$$\left. + \frac{aA \exp(-b\alpha_*)}{b^2 + \omega_{ac}^2}(\omega_{ac} \sin \omega_{ac}\alpha_* - b \cos \omega_{ac}\alpha_*)\right]. \tag{15}$$

Here α_* and β_* depend on ξ.

Relations (14), (15) determine the law of motion of the system $x(t)$ at specified linearly independent functions $x_k(t)$, $k = 1, 2$, and experimental parameters.

From (11), (12) we find

$$\ddot{x}(t) = -\frac{P_{st}}{M} + \frac{c_s A}{M}\left(1 - \frac{x(t)}{2A}\right)^{\alpha}. \tag{16}$$

We also denote $y = 1 - \frac{x}{2A}$; thus (16) is written as

$$\ddot{y} = \frac{P_{st}}{2AM} - \frac{c}{2M} y^{\alpha} .$$

The first integral of the equation has the form

$$\dot{y}^2 = \frac{P_{st}}{2AM} y - \frac{c y^{\alpha+1}}{M(1+\alpha)} + C . \tag{17}$$

The quantity C is found from the initial ($t = 0$) conditions

$$x(0) = 2A\ ,\quad y(0) = 1 - \frac{x(0)}{2A} = 0\ ,\quad \dot{x}(0) = -v_0\ ,\quad \dot{y}(0) = -\frac{\dot{x}(0)}{2A} = \frac{v_0}{2A}\ .$$

We present the differential equation (17) and the initial conditions relative to the unknown function $y(t)$ in the form

$$\dot{y}^2 = h^2(y)\ ,\quad h(y) = (d + by - ay^{\alpha+1})^{1/2} \quad (0 < t < t_{**})\ , \tag{18}$$

$$y(0) = 0\ ;\quad a = \frac{c}{M(1+\alpha)}\ ;\quad b = \frac{P_{st}}{AM}\ ;\quad d = \frac{v_0^2}{4A^2}\ .$$

The region of determination of the function $h(y)$ satisfies the inequalities

$$0 \le y \le 1\ ,\quad d + by - ay^{\alpha+1} \ge 0\ ,\quad a, b, d \ge 0\ , \tag{19}$$

and, as can be easily seen, it depends on the value of the constant

$$\Delta = d + b - a = \frac{v_0^2}{4A^2} + \frac{P_{st}}{AM} - \frac{c}{M(1+\alpha)}\ . \tag{20}$$

We denote the only positive root of the equation $h(y) = 0$ in terms of y_*. When $\Delta > 0$, we have $y_* > 1$ and the section $y \in [0, 1]$ is the region of determination of $h(y)$. If $\Delta < 0$, then $y_* < 1$ and the the section $0 \le y \le y_*$ is the region of determination. The boundary case appears at $\Delta = 0$, $y_* = 1$, $0 \le y \le 1$. For the indicated ratios of the parameters it is convenient to present the behavior of system (18) by the phase picture of motion in the variables $x = 2A(1 - y)$, $v = \dot{x} = -2A\dot{y}$ (Fig. 3). According to (16), when $\Delta > 0$ the prevailing effect on motion of the system is exerted by the static parameters — force P_{st} and the initial velocity (at the instant when the vibrocontact between the system and the base enters phase I) of the system v_0, when the contribution of vibroshock interaction is relatively small. In this case, the phase trajectory follows the curve which originates at $t = 0$ from the point A_1 ($x = 2A$, $v = -v_0$) and approaches the point $A_2'(0, -v_1)$ at $t = t_*$. Since $v_1 > 0$, elastic collision of the system with the base takes place, which instantaneously (at $t = t_*$) changes the velocity of the system to the opposite (point $A_2''(0, v_1)$). Then the vibrosystem moves away from the base, being first (when $t_* < t \le t_{**}$) in phase I ($0 \le x \le 2A$) of the vibrocontact with the base on the section $A_2''A_3$ of the phase trajectory 1. When $t > t_{**}$, the system moves beyond phase I under the action of static force P_{st} only, which by the end of the period of motion returns the system to the contact zone (section $A_3A_4A_1$ of trajectory 1).

When $\Delta < 0$, the system follows the phase trajectory 2. In this case, according to (16), the prevailing contribution in phase I is made by the momentum due to vibrocontact; the system is smoothly retarded at the instant $t_*(\dot{x}(t_*) = 0)$ at the point $x(t_*) = x_*$ which is at a distance from the base ($x_* > 0$). Here, the phase trajectory 2 is continuous at a point similar to point A_2 of phase trajectory 1 (points A_2' and A_2'' coincide with the latter).

The intermediate phase trajectory 3 corresponds to $\Delta = 0$. In this case, at $t = t_*$ the system touches the base $x = 0$ at the zero velocity $\dot{x}(t_*)$. Subsequent motion, as in the previous case, follows behind the section $A_2''A_3A_4A_1$ of the phase trajectory 1.

We note that the system studied possesses singularity L (of the center type) with the coordinates $x = 2A\left[1 - \left(\frac{P_{st}}{Ac}\right)^{1/\alpha}\right]$, $\dot{x} = 0$, which is the point of libration. It follows from (18) that

$$\dot{y} = h(y) \quad (0 < t < t_*)\ ,\quad \dot{y} = -h(y) \quad (t_* < t < t_{**})\ ,\quad y(0) = 0\ . \tag{21}$$

The solution of the Cauchy problem (21) is written as

$$t = \int_0^y \frac{dz}{h(z)} \quad (0 < t < t_*) , \quad t = t_* - \int_{z_*}^y \frac{dz}{h(z)} \quad (t_* < t < t_{**}) , \tag{22}$$

$$z_* = \begin{cases} y_* , & y_* < 1 ; \\ 1 , & y_* \geq 1 . \end{cases}$$

Relations (22) determine the unknown function $y(t)$ and the law of motion of the system $x(t)$ at the given dependence $h(t)$, e.g., in the form of (18). Values of t_* and t_{**} are found by the formulas

$$t_* = \int_0^{z_*} \frac{dz}{h(z)} , \quad t_{**} = 2t_* , \tag{23}$$

the second formula follows from the symmetry of problem (21).

With account for (11), (12), the momentum imparted to the base is

$$I = 2cA \int_0^{t_*} \left[1 - \frac{x(t)}{2A} \right]^{\alpha} dt = 2cA \int [y(t)]^{\alpha} dt ; \tag{24}$$

here $y(t)$ is determined by the first formula of (22).

Motion of the system in phase II beyond vibrocontact occurs according to the equations

$$M\ddot{x}(t) = -P_{\text{st}} \quad (t_{**} < t < t_{\text{s}}) , \quad x(t_{**}) = 2A , \quad \dot{x}(t_{**}) = v_0 , \tag{25}$$

whence

$$x(t) = -\frac{P_{\text{st}}}{2M} (t - t_{**})^2 + v_0 (t - t_*) + 2A .$$

The time of motion in phase II satisfies the condition $x(t_{\text{s}}) = 2A$, therefore

$$t_{\text{s}} - t_{**} = \frac{2Mv_0}{P_{\text{st}}} .$$

The period of vibrosystem motion t_{s} per cycle is

$$t_{\text{s}} = t_{**} + \frac{2Mv_0}{P_{\text{st}}} = 2 \left(t_* + \frac{Mv_0}{P_{\text{st}}} \right), \tag{26}$$

where t_* is found from (23).

With the aim of illustration, we consider small vibrations of the vibroacoustic system (Fig. 4). The equation of motion of the pneumatic knock-out near the base O is written as

$$J\ddot{\varphi} = \frac{F_1 l_1 \left(1 + \frac{1}{\cos \varphi} \right)}{2} - \frac{F_2 l_2}{2} - Pl_4 + Rl_3 - Sl_3 - Ql_1 , \tag{27}$$

here

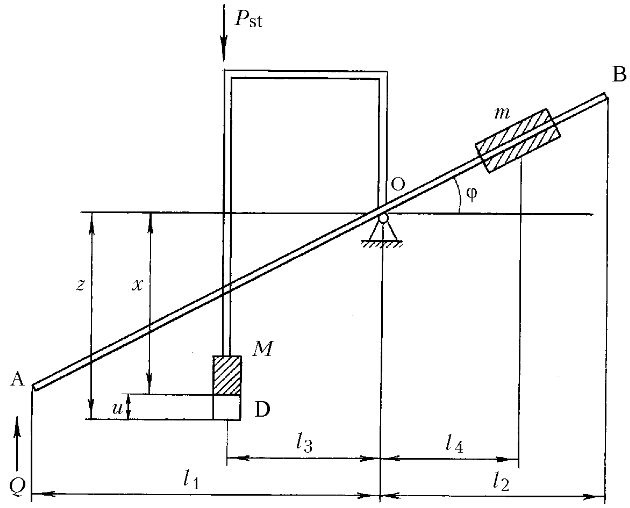

Fig. 4. Model of the vibroacoustic system.

$$J = 1/3\gamma\,(l_1^3 + l_2^3) + Ml_3^2\,; \quad F_1 = \gamma l_1 g\,; \quad F_2 = \gamma l_2 g\,; \quad P = mg\,.$$

For the contact force of interaction between the vibrator and pneumatic knock-out at the point D we take the averaged relation

$$R = \begin{cases} cA\left(1 - \dfrac{z-x}{2A}\right)^{\alpha}, & 0 \le z - x \le 2A\,; \\ 0\,, & z - x > 2A\,. \end{cases} \tag{28}$$

Here, the relations $u = z - x$ and $u < z - x$ (in the presence and absence of the contact, respectively) are taken into account.

The form of force S is determined by the character of damping of the pneumatic knock-out, for example, for viscoelastic steel it can be taken as:

$$S = k_1\,(z - z_*)^{\beta_1} + k_2 \dot{z}^{\,\gamma_1} \tag{29}$$

with the parameters k_1, k_2, β_1, and γ_1 chosen experimentally.

The form of the function Q is determined by the mechanical properties of the material and the shape of the contacting part being processed.

We write the equation of motion of mass M with no regard for the weight of the fixture of the vibrator to the pneumatic knock-out:

$$M\ddot{x} = P_{st} - R\,. \tag{30}$$

Since the vibrations of the pneumatic knock-out are small, we assume $z = l_3\varphi$ and rewrite Eq. (27) in the form

$$\ddot{z} = N + (R - S)\,\frac{l_3^2}{J} - \frac{Ql_1 l_3}{J}\,, \tag{31}$$

where $N = 0.5F_1 l_1 - F_2 l_2 - Pl_4$.

At the given dependences $Q(z, \dot{z})$, $R(x, z)$, and $S(z, \dot{z})$ the system of differential equations (30), (31) with initial conditions

$$x(0) = x_0 , \quad \dot{x}(0) = v_0 , \quad z(0) = z_0 , \quad \dot{z}(0) = v_1 \tag{32}$$

determines the nonlinear Cauchy problem for the sought-for functions $x(t)$, $z(t)$. To qualitatively analyze this differential model, we thoroughly consider the main case where the strength of the part processed is high and the displacement of the working end A of the pneumatic knock-out during one multiple interaction of the end with the part can be neglected. Assuming $z(t) = z_0$, we take the force Q on the working end, which is necessary to provide this regime, to be unknown and determine it together with the function $x(t)$ from (30)–(32). We introduce the functions

$$h = 1 - \frac{z_0 - x}{2A} , \quad R = cAh^{\alpha} \quad (h \ge 0) . \tag{33}$$

From (30) we obtain the equation with respect to $h(t)$

$$\ddot{h} = \frac{P_{\text{st}}}{2AM} - \frac{ch^{\alpha}}{2M} . \tag{34}$$

The first integral of this equation is

$$\dot{h}^2 = \frac{P_{\text{st}} h}{AM} - \frac{ch^{\alpha+1}}{M(\alpha + 1)} + a_1 .$$

From initial conditions (32) with account for (33) and the relations $u(0) = z_0$, $h(0) = 1$, and $\dot{h}(0) = \dot{x}(0)/(2A) = v_0/(2A)$ we find the constant

$$a_1 = \left(\frac{v_0}{2A}\right)^2 - \frac{P_{\text{st}}}{AM} + \frac{c}{M(\alpha + 1)} . \tag{35}$$

For brevity, we write Eq. (34) as

$$\dot{h}^2 = a_1 + a_2 h - a_3 h^{\alpha+1} \quad \left(a_2 = \frac{P_{\text{st}}}{AM} , \quad a_3 = \frac{c}{M(\alpha + 1)}\right). \tag{36}$$

Allowing for the fact that $\dot{h}(t) = \dot{x}(t)/(2A) > 0$, $h(t) \ge 1$, we find the solution of Eq. (36) in the form

$$t = \int_1^h \frac{d\xi}{\sqrt{H(\xi)}} , \tag{37}$$

where $H(\xi) = a_1 + a_2\xi - a_3\xi^{\alpha+1}$.

Integral (37) is determined (in the improper sense) on $h \in [1; h_1]$, where $H(h) = a_1 + a_2 h - a_3 h^{\alpha+1} \ge 0$. Thus, at $\alpha = 1$

$$\alpha_1 = (a_2 + \sqrt{a_2^2 + 4a_1 a_2})/(2a_3) .$$

According to (33) and with account for (37), the sought-for function $x(t)$ has the form

$$x(t) = 2A\,[h(t) - 1] + z_0 . \tag{38}$$

From (31) and (29) we find the cutting force as

$$Q(t) = \frac{J}{l_1 l_3}\left[N + (cAh^{\alpha}(\alpha) - S)\frac{l_3^2}{J}\right], \tag{39}$$

$$S = k_1 (z_0 - z_*)^{\beta} + k_2 v_1^{\gamma}; \quad N = \frac{1}{2}(2F_1 l_1 - F_2 l_2) - P_{st} l_1 .$$

Thus, as a result of investigations made we described the mathematical model of the open-loop vibroacoustic system of the modernized sawing section of the ShP-2 mill. To determine the character of interaction between the workpiece and the cutting tool, we suggested a special nonlinear law of vibrations of the sawing boom, which allows more thorough description of the process of interaction between the end of the rod-concentrator of the ultrasonic transformer with the rigid base of the adjustment support.

NOTATION

A, amplitude of harmonic ultrasonic vibration, m; a, b, a_0, b_0, a_1, a_2, a_3, b_1, c, c_1, d, A_k, k_1, k_2, α, α_1, β_1, γ_1, coefficients determined experimentally; C, integration constant; c_s, rigidity of the system, N/m; $F(t)$, $F(x/2A)$, monotonically decreasing functions; $f(t)$, law of vibrations of an unloaded end (at $R = 0$); F_1, F_2, F_3, P, weight of different parts of the system, N; $G(\tau, \xi)$, $\Phi(\xi)$, integrands determining the law of motion of the vibroacoustic system; g, free-fall acceleration, m/sec^2; $h(y)$, function depending on the position of the mass of the vibroelement; h_1, first positive real root of the equation $H(h) = 0$; $I(t)$, momentum imparted by the vibroacoustic system to the base, kg·m/sec; I_{ac}, mean value of the momentum during vibroshock contact; kg·m/sec; I_m, mean value of the momentum during ultrasonic vibration, kg·m/sec; J, moment of inertia of the system relative to point O, m^4; l_1, l_2, l_3, l_4, distances to different parts of the system, m; m, mass of the balance, kg; M, concentrated mass of the acoustic vibrator, kg; P_{st}, static force of the vibrator press, N; Q, force affecting the workpiece being processed, N; R, contact force of interaction between the end of the vibrator and the pneumatic knock-out, N; $R(t)$, force at the end of the element in contact with the base ($R \geq 0$), N; S, reaction of the damping element, N; t, time of motion of the vibroacoustic system, sec; t_*, instant of elastic collision of the system with the base, sec; t_{**}, time of transition of the system to the phase of motion under the action of static force, sec; t_s, time of motion of the vibrosystem, sec; T_{ac}, period of harmonic ultrasonic vibration, sec; T_s, duration of vibroshock contact, sec; $u(t)$, displacement of the end of the element relative to the position $x(t)$ of the mass M, m; v, speed of the vibroacoustic system, m/sec; v_0, initial value of vibroacoustic speed of the system, m/sec; v_1, value of the speed of the vibroacoustic system at the instant of contact with the base, m/sec; x, instantaneous value of the coordinate of the position of the mass of the vibroelement, m; $x(t)$, coordinate of the position of the mass of the vibroelement, m; x_0, initial value of the coordinate of the position of the mass of the vibroelement, m; $x_k(t)$, specified linearly independent functions; y, function depending on the position of mass of the vibroelement; y_*, the only positive root of the equation $h(y) = 0$; z, vertical displacement of the system, mm; z_0, initial value of vertical displacement of the system, m; z_*, value of the lower limit of integration depending on y_*, m; α_s and β_s, upper and lower time limits of a single collision, sec; $\alpha_s(x)$ and $\beta_s(x)$, found values of the upper and lower time limits of a single collision; γ, mass per unit length of the pneumatic knock-out, kg/m; $\gamma(t)$, monotonically decreasing function; Γ, set of time intervals of the contact, when $R(t) \geq 0$; Δ, constant determining the character of motion of the vibroacoustic system; ξ, ϑ, τ, integration variables; φ, small angle of deviation of the pneumatic knock-out, deg; ν_{ac}, frequency of harmonic ultrasonic vibration, Hz; ω_{ac}, angular frequency of harmonic ultrasonic vibration, Hz. Subscripts: ac, acoustic vibrations; s, system; m, mean; st, static; k, index for enumeration of linearly independent functions.

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